'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 4th Semester Examination, 2023

GE2-P2-STATISTICS

FUNDAMENTAL OF PROBABILITY THEORY

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.

GROUP-A

1. Answer any ***five*** questions: 1×5 = 5

(a) If $P(A \cup B) = \frac{5}{6}$, $P(A \cap B) = \frac{1}{3}$ and $P(A^c) = \frac{1}{2}$, then show that A and B are independent.

(b) What is the chance that a non-leap year selected at random will contain 53 Sundays?

(c) The mean and variance of a binomial variate X are 4 and $\frac{4}{3}$. Find $P(X = 1)$.

(d) Show that the chance of throwing an odd number with a die is $\frac{1}{2}$.

(e) For any random variable X, show that $\text{var}(a - bX) = b^2 \text{var}(X)$.

(f) For any two events A and B, show that $P(A + B) \le P(A) + P(B)$.

(g) Explain discrete probability distribution.

(h) Distinguish between p.m.f and p.d.f.

GROUP-B

2. Answer any ***three*** questions: 5×3 = 15

(a) State and prove Bayes' Theorem.

(b) Derive Poisson distribution as the limit of binomial distribution.

(c) Find the variance of binomial distribution.

(d) If X has Poisson distribution with parameter λ, then show that

$$P[X \text{ is even}] = \frac{1}{2}[1 + e^{-2\lambda}]$$

(e) The mean of a normal distribution is 50 and 5% of the values are greater than 60. Find the s.d. of the distribution. (Given that the area under standard normal curve between $Z = 0$ and $Z = 1.64$ is 0.45).

GROUP-C

3. Answer any ***two*** questions: 10×2 = 20

(a) Find the mean and variance of normal distribution.

(b) (i) Show that the expectation of the sum of two jointly distributed random variable X and Y is the sum of their expectations.

(ii) For what value of k, $f(x, y)$ represents the probability density function of two continuous random variable X and Y?

$$f(x, y) = k(4 - 2x + y), \quad 0 < x < 3, \quad 2 < y < 4$$
$$= 0, \text{ elsewhere}$$

(c) (i) Let the variable X have the distribution $P(X = 0) = P(X = 2) = p$, $P(X = 1) = 1 - 2p$ for $0 \le p \le \frac{1}{2}$. For what value of p is the var(X) maximum?

(ii) Find the mode of the binomial distribution.

(d) (i) Find the probability that at most 5 defective fuses will be found in a box of 200 fuses, if experience show that 2% of such fuses are defective.

(ii) A coin is tossed until a head appears. What is the expectation of the number of tosses?

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